两角和与差公式、二倍角公式答案与解析

1. B 【解析】本题考查二倍角的正弦公式. 因为角 α 的终边位于直线 $2x-y=0$ 上,所以 $\tan\alpha=2$, $\sin 2\alpha=2\sin\alpha\cos\alpha=\frac{2\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha}=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{4}{1+4}=\frac{4}{5}$,故选 B.

2. D 【解析】本题考查两角差的余弦公式、同角三角函数基本关系式. 因为 α 为第二象限角, $\sin\alpha=\frac{1}{3}$,所以 $\cos\alpha=-\frac{2\sqrt{2}}{3}$,因此 $\cos\left(\alpha-\frac{\pi}{4}\right)=\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}=-\frac{2\sqrt{2}}{3}\times\frac{\sqrt{2}}{2}+\frac{1}{3}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}-4}{6}$,故选 D.

3. B 【解析】本题考查同角三角函数基本关系、二倍角公式及两角和与差的正弦公式.

$\because\cos\alpha+\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)=0$, $\therefore\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)+\frac{\sqrt{2}}{2}\cdot\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)=0$,即 $\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}+\frac{\sqrt{2}}{2}\right)=0$.

$\because\alpha\in(\pi,2\pi)$, $\therefore\frac{\alpha}{2}\in\left(\frac{\pi}{2},\pi\right)$, $\therefore\cos\frac{\alpha}{2}\neq\sin\frac{\alpha}{2}$, $\therefore\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}+\frac{\sqrt{2}}{2}=0$, $\therefore\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}=-\frac{\sqrt{2}}{2}<0$, $\therefore\frac{\alpha}{2}\in\left(\frac{3\pi}{4},\pi\right)$,上式两边平方得 $1+\sin\alpha=\frac{1}{2}$, $\therefore\sin\alpha=-\frac{1}{2}$, $\because\alpha\in\left(\frac{3\pi}{2},2\pi\right)$,

$\therefore\cos\alpha=\frac{\sqrt{3}}{2}$,则 $\sin\left(\alpha+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\sin\alpha+\frac{1}{2}\cos\alpha=0$. 故选 B.

4. C 【解析】本题考查诱导公式及余弦的和差公式的应用. $\cos 67°\cos 52°+\cos 23°\cos 38°=\sin 23°\sin 38°+\cos 23°\cos 38°=\cos(38°-23°)=\cos 15°=\cos(45°-30°)=\cos 45°\cos 30°+\sin 45°\sin 30°=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}$,故选 C.

5. A 【解析】本题考查诱导公式、二倍角公式及两角和的正弦公式.

方法一:由 $2\cos 2\alpha=\sin\left(\frac{\pi}{4}+\alpha\right)$,得 $2(\cos^2\alpha-\sin^2\alpha)=\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)$. 因为 $\alpha\in\left(\pi,\frac{3\pi}{2}\right)$,所以 $\sin\alpha+\cos\alpha\neq0$,于是 $\cos\alpha-\sin\alpha=\frac{\sqrt{2}}{4}$,两边平方得 $1-\sin 2\alpha=\frac{1}{8}$,所以 $\sin 2\alpha=\frac{7}{8}$. 故选 A.

方法二:由 $2\cos 2\alpha=\sin\left(\frac{\pi}{4}+\alpha\right)$,得 $2\sin\left(\frac{\pi}{2}+2\alpha\right)=\sin\left(\frac{\pi}{4}+\alpha\right)$,所以 $4\sin\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}+\alpha\right)=\sin\left(\frac{\pi}{4}+\alpha\right)$. 因为 $\alpha\in\left(\pi,\frac{3\pi}{2}\right)$,所以 $\frac{\pi}{4}+\alpha\in\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$,所以 $\sin\left(\frac{\pi}{4}+\alpha\right)<0$,所以 $\cos\left(\frac{\pi}{4}+\alpha\right)=\frac{1}{4}$, $\sin\left(\frac{\pi}{4}+\alpha\right)=-\frac{\sqrt{15}}{4}$,所以 $\sin 2\alpha=-\cos\left(\frac{\pi}{2}+2\alpha\right)=-\left[\cos^2\left(\frac{\pi}{4}+\alpha\right)-\sin^2\left(\frac{\pi}{4}+\alpha\right)\right]=-\left(\frac{1}{16}-\frac{15}{16}\right)=\frac{7}{8}$.

6. D 【解析】本题考查诱导公式及二倍角公式. 因为 $2\alpha-\frac{\pi}{3}=2\left(\alpha+\frac{\pi}{3}\right)-\pi$,所以 $\cos\left(2\alpha-\frac{\pi}{3}\right)=\cos\left[2\left(\alpha+\frac{\pi}{3}\right)-\pi\right]=-\cos\left[2\left(\alpha+\frac{\pi}{3}\right)\right]=-\left[1-2\sin^2\left(\alpha+\frac{\pi}{3}\right)\right]=2\times\left(\frac{\sqrt{3}}{3}\right)^2-1=-\frac{1}{3}$. 故选 D.

7. B 【解析】本题考查和差角公式、倍角公式及辅助角公式的应用. $a=\frac{1}{\sqrt{2}}(\sin 56°-\cos 56°)=\frac{1}{\sqrt{2}}\times\sqrt{2}\sin(56°-45°)=\sin 11°=\cos 79°$, $b=\cos 50°\cos 128°+\cos 40°\cos 38°=-\cos 50°\cos 52°+\sin 50°\cdot\sin 52°=-\cos 102°=\cos 78°$, $c=\frac{1}{2}(\cos 80°-2\cos^2 50°+1)=\frac{1}{2}(\cos 80°-\cos 100°)=\frac{1}{2}(\cos 80°+\cos 80°)=\cos 80°$. $\because\cos 78°>\cos 79°>\cos 80°$, $\therefore b>a>c$,故选 B.

8. B 【解析】本题考查和差角公式的应用. 由 $\alpha\in\left[\pi,\frac{3\pi}{2}\right]$, $\beta\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$,得 $\frac{\pi}{2}\leqslant\alpha-\beta\leqslant\frac{5\pi}{4}$, $\frac{5\pi}{4}\leqslant\alpha+\beta\leqslant2\pi$. $\because\sin(\alpha-\beta)=\frac{\sqrt{10}}{10}$, $\therefore\frac{\pi}{2}\leqslant\alpha-\beta\leqslant\pi$, $\therefore\cos(\alpha-\beta)=-\sqrt{1-\left(\frac{\sqrt{10}}{10}\right)^2}=-\frac{3\sqrt{10}}{10}$.

由 $\beta\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ 得 $2\beta\in\left[\frac{\pi}{2},\pi\right]$, $\therefore\cos 2\beta=-\sqrt{1-\left(\frac{\sqrt{5}}{5}\right)^2}=-\frac{2\sqrt{5}}{5}$,而 $\cos(\alpha+\beta)=\cos[(\alpha-\beta)+2\beta]=\cos(\alpha-\beta)\cos 2\beta-\sin(\alpha-\beta)\sin 2\beta=-\frac{3\sqrt{10}}{10}\times\left(-\frac{2\sqrt{5}}{5}\right)-\frac{\sqrt{10}}{10}\times\frac{\sqrt{5}}{5}=\frac{\sqrt{2}}{2}$. 又 $\frac{5\pi}{4}\leqslant\alpha+\beta\leqslant2\pi$, $\therefore\alpha+\beta=\frac{7\pi}{4}$,故选 B.

9. B 【解析】本题考查半角的正切公式、两角和的正切公式. 设 $BC=x$,则 $AC=x+1$. $\because AB=5$, $\therefore 5^2+x^2=(x+1)^2$, $\therefore x=12$,即水深为 12 尺,芦苇长为 13 尺.

$\because\tan\theta=\frac{BC}{AB}=\frac{12}{5}$, $\therefore\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{12}{5}$,解得 $\tan\frac{\theta}{2}=\frac{2}{3}$(负根舍去).

$\because\tan\theta=\frac{12}{5}$, $\therefore\tan\left(\theta+\frac{\pi}{4}\right)=\frac{1+\tan\theta}{1-\tan\theta}=-\frac{17}{7}$. 故正确结论的编号为①③④. 故选 B.

10. BD 【解析】本题考查二倍角公式. $\sin 15°\cos 15°=\frac{\sin 30°}{2}=\frac{1}{4}$,排除 A; $\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}=\cos\frac{\pi}{3}=\frac{1}{2}$,B 正确; $\sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}=\frac{\sqrt{2+\sqrt{3}}}{2}$,排除 C;由 $\tan 45°=\frac{2\tan 22.5°}{1-\tan^2 22.5°}$,得 $\frac{\tan 22.5°}{1-\tan^2 22.5°}=\frac{1}{2}$,D 正确. 故选 BD.

11. BD 【解析】本题考查二倍角公式及同角基本关系. $\because 3\pi\leqslant\theta\leqslant4\pi$, $\therefore\frac{3\pi}{2}\leqslant\frac{\theta}{2}\leqslant2\pi$, $\therefore\cos\frac{\theta}{2}\geqslant0$, $\sin\frac{\theta}{2}\leqslant0$,则 $\sqrt{\frac{1+\cos\theta}{2}}+\sqrt{\frac{1-\cos\theta}{2}}=\sqrt{\cos^2\frac{\theta}{2}}+\sqrt{\sin^2\frac{\theta}{2}}=\cos\frac{\theta}{2}-\sin\frac{\theta}{2}=\sqrt{2}\cdot\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=\frac{\sqrt{6}}{2}$, $\therefore\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$, $\therefore\frac{\theta}{2}+\frac{\pi}{4}=\frac{\pi}{6}+2k\pi(k\in\mathbf{Z})$ 或 $\frac{\theta}{2}+\frac{\pi}{4}=-\frac{\pi}{6}+2k\pi(k\in\mathbf{Z})$,即 $\theta=-\frac{\pi}{6}+4k\pi(k\in\mathbf{Z})$ 或 $\theta=-\frac{5\pi}{6}+4k\pi(k\in\mathbf{Z})$,又 $3\pi\leqslant\theta\leqslant4\pi$, $\therefore\theta=\frac{19\pi}{6}$ 或 $\frac{23\pi}{6}$. 故选 BD.

12. AC 【解析】本题考查平方关系式、两角差的余弦公式的应用.
由已知,得 $\sin\gamma=\sin\beta-\sin\alpha$, $\cos\gamma=\cos\alpha-\cos\beta$,
两式分别平方相加,得 $(\sin\beta-\sin\alpha)^2+(\cos\alpha-\cos\beta)^2=1$,
$\therefore -2\cos(\beta-\alpha)=-1$, $\therefore\cos(\beta-\alpha)=\frac{1}{2}$, $\therefore$ A 正确,B 错误.

$\because\alpha,\beta,\gamma\in\left(0,\frac{\pi}{2}\right)$, $\therefore\sin\gamma=\sin\beta-\sin\alpha>0$, $\therefore\beta>\alpha$,

$\therefore\beta-\alpha=\frac{\pi}{3}$, $\therefore$ C 正确,D 错误. 故选 AC.

13. $\frac{4}{3}$ **【解析】**本题考查同角三角函数基本关系及二倍角公式. 因为 $\cos\theta=-\frac{\sqrt{5}}{5}$且$\theta\in\left(\frac{\pi}{2},\pi\right)$，所以 $\sin\theta=\frac{2\sqrt{5}}{5}$，则 $\tan\theta=-2$，所以 $\tan 2\theta=\frac{2\times(-2)}{1-(-2)^2}=\frac{4}{3}$.

14. $-\frac{4}{5}$ **【解析】**本题考查同角基本关系、诱导公式及二倍角公式. $\cos\left(\frac{\pi}{2}+2\alpha\right)=-\sin 2\alpha=\frac{-2\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha}=\frac{-2\tan\alpha}{\tan^2\alpha+1}=\frac{-2\times2}{2^2+1}=-\frac{4}{5}$.

15. $-\frac{\sqrt{3}}{2}$ **【解析】**本题考查和差角公式的逆用. $\cos 10°-2\cos 20°\cdot\cos 10°=\cos(20°-10°)-2\cos 20°\cos 10°=\cos 20°\cos 10°+\sin 20°\sin 10°-2\cos 20°\cos 10°=\sin 20°\sin 10°-\cos 20°\cos 10°=-\cos(20°+10°)=-\cos 30°=-\frac{\sqrt{3}}{2}$.

16. $-\frac{\sqrt{3}}{3}$ $-\sqrt{2}$ **【解析】**本题考查诱导公式和同角三角函数基本关系的应用. 由题意 $\sin\left(\theta-\frac{2\pi}{3}\right)=-\sin\left(\theta-\frac{2\pi}{3}+\pi\right)=-\sin\left(\theta+\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{3}$，$\cos\left(\theta-\frac{\pi}{6}\right)=\sin\left[\left(\theta-\frac{\pi}{6}\right)+\frac{\pi}{2}\right]=\sin\left(\theta+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{3}$，$\therefore\theta-\frac{\pi}{6}$是第一或第四象限角，$\because\theta$是第四象限角，$\therefore\theta-\frac{\pi}{6}$是第四象限角，$\therefore\sin\left(\theta-\frac{\pi}{6}\right)=-\sqrt{1-\cos^2\left(\theta-\frac{\pi}{6}\right)}=-\sqrt{1-\left(\frac{\sqrt{3}}{3}\right)^2}=-\frac{\sqrt{6}}{3}$，$\therefore\tan\left(\theta-\frac{7\pi}{6}\right)=\tan\left(\theta-\frac{\pi}{6}\right)=\frac{\sin\left(\theta-\frac{\pi}{6}\right)}{\cos\left(\theta-\frac{\pi}{6}\right)}=\frac{-\frac{\sqrt{6}}{3}}{\frac{\sqrt{3}}{3}}=-\sqrt{2}$.

17. $\frac{1}{3}$ **【解析】**本题考查诱导公式、二倍角公式及两角和的正弦公式.

方法一：$f(x)=\cos x(\sin x+\cos x)-\frac{1}{2}=\sin x\cos x+\cos^2x-\frac{1}{2}=\frac{1}{2}\sin 2x+\frac{1+\cos 2x}{2}-\frac{1}{2}=\frac{1}{2}\sin 2x+\frac{1}{2}\cos 2x=\frac{\sqrt{2}}{2}\cdot\sin\left(2x+\frac{\pi}{4}\right)$，因为 $f(\alpha)=\frac{\sqrt{2}}{6}$，所以 $\sin\left(2\alpha+\frac{\pi}{4}\right)=\frac{1}{3}$. 所以 $\cos\left(\frac{\pi}{4}-2\alpha\right)=\cos\left[\frac{\pi}{2}-\left(2\alpha+\frac{\pi}{4}\right)\right]=\sin\left(2\alpha+\frac{\pi}{4}\right)=\frac{1}{3}$.

方法二：$f(x)=\cos x(\sin x+\cos x)-\frac{1}{2}=\sin x\cos x+\cos^2x-\frac{1}{2}=\frac{1}{2}\sin 2x+\frac{1+\cos 2x}{2}-\frac{1}{2}=\frac{1}{2}\sin 2x+\frac{1}{2}\cos 2x$. 因为$f(\alpha)=\frac{\sqrt{2}}{6}$，所以 $\sin 2\alpha+\cos 2\alpha=\frac{\sqrt{2}}{3}$. 所以 $\cos\left(\frac{\pi}{4}-2\alpha\right)=\cos\frac{\pi}{4}\cdot\cos 2\alpha+\sin\frac{\pi}{4}\sin 2\alpha=\frac{\sqrt{2}}{2}(\cos 2\alpha+\sin 2\alpha)=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{3}=\frac{1}{3}$.

18.【解】(1)$\because\alpha,\beta$为锐角，且 $\cos\beta=\frac{\sqrt{5}}{5}$，$\cos(\alpha+\beta)=-\frac{\sqrt{5}}{5}$，

$\therefore\alpha+\beta\in\left(\frac{\pi}{2},\pi\right)$，$\therefore\sin\beta=\sqrt{1-\cos^2\beta}=\frac{2\sqrt{5}}{5}$，

$\sin(\alpha+\beta)=\sqrt{1-\cos^2(\alpha+\beta)}=\frac{2\sqrt{5}}{5}$，

$\therefore\cos\alpha=\cos[(\alpha+\beta)-\beta]=\cos(\alpha+\beta)\cos\beta+\sin(\alpha+\beta)\sin\beta=\frac{3}{5}$，$\therefore\cos 2\alpha=2\cos^2\alpha-1=-\frac{7}{25}$.

(2)由(1)得 $\cos\alpha=\frac{3}{5}$，

则 $\sin\alpha=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$，$\tan\alpha=\frac{4}{3}$.

又 $\cos\beta=\frac{\sqrt{5}}{5}$，$\sin\beta=\frac{2\sqrt{5}}{5}$，$\therefore\tan\beta=2$.

$\therefore\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=-\frac{2}{11}$.