

【第 22 题】

1、知识与能力要求：理解 n 次独立重复试验的模型（ n 重伯努利试验）及其意义 理解二项分布，并能解决一些简单的实际问题

2、主要存在问题及错因

最大的问题是：是学生对题意得理解出现了问题：没有理解题目中所说的“开每扇门相互独立”，把每扇门都“给定的 6 把钥匙”误以为总共就 6 把钥匙。所以导致第（1）（2）问均有不同程度的失分。

第一问：本小题相对而言得分尚可，错误主要在两方面：（1）未能准确反映“若连续 4 次未能打开，则放弃这扇门”，将独立重复试验误解成了前后两把锁是相互有影响的，即 6 把钥匙要连续使用而不是每把锁都是相互独立的 6 把钥匙；（2）未能验证概率和为 1 导致求解中的错误未能及时修正。

第二问：得分相对较低，主要由第一问的错误的后续造成，当然未能正确求出打开 1 把锁的概率是重灾区，也有计算错误及独立重复试验的掌握不准确导致的列式错误。

典型错例

The image shows two pages of handwritten student work for a probability problem. The problem involves 4 doors, each with 6 keys, and a student trying to open them. The student's solutions contain several errors.

Left Page (Question 22):

- Assumes $X=1, 2, 3, 4$ for the number of keys tried.
- Calculates $P(X=1) = \frac{1}{6}$, $P(X=2) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$, $P(X=3) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$, and $P(X=4) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{360}$.
- Constructs a probability distribution table for X :

X	1	2	3	4
$P(X)$	$\frac{1}{6}$	$\frac{1}{30}$	$\frac{1}{120}$	$\frac{1}{360}$

- Calculates the expected value $E(X) = \frac{179}{360}$.
- For the second question, calculates the probability of opening a door as $\frac{1}{6} + \frac{1}{30} + \frac{1}{120} + \frac{1}{360} = \frac{71}{360}$.
- Calculates the probability of not opening a door as $P = \left(\frac{5}{6}\right)^4 \times \left(1 - \frac{71}{360}\right)$.

Right Page (Question 22):

- Assumes $X=1, 2, 3, 4$ for the number of keys tried.
- Calculates $P(X=1) = \frac{1}{6}$, $P(X=2) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$, $P(X=3) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$, and $P(X=4) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{360}$.
- Constructs a probability distribution table for X :

X	1	2	3	4
$P(X)$	$\frac{1}{6}$	$\frac{1}{30}$	$\frac{1}{120}$	$\frac{1}{360}$

- Calculates the expected value $E(X) = \frac{179}{360}$.
- For the second question, calculates the probability of opening a door as $\frac{1}{6} + \frac{1}{30} + \frac{1}{120} + \frac{1}{360} = \frac{71}{360}$.
- Calculates the probability of not opening a door as $P = \left(\frac{5}{6}\right)^4 \times \left(1 - \frac{71}{360}\right)$.

22. 【必做题】

(1) 解: 由题意知, X 可取值为 1, 2, 3, 4

$P(X=1) = \frac{1}{6}$

$P(X=2) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$

$P(X=3) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$

$P(X=4) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times (\frac{1}{3} + \frac{1}{5}) = \frac{1}{120}$

列表如下:

X	1	2	3	4
P	$\frac{1}{6}$	$\frac{1}{30}$	$\frac{1}{120}$	$\frac{1}{120}$

$\therefore E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{30} + 3 \times \frac{1}{120} + 4 \times \frac{1}{120} = \frac{7}{24}$

$\therefore X$ 的分布列如表所示, 其数学期望 $E(X) = \frac{7}{24}$

(2) 记 4 次成功打开 4 扇门为事件 A

连续 4 次都成功

从第 1 扇开始, 4 次都成功 $P = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{360}$

共 3 次, 恰有 4 次成功

$P_2 = \frac{5}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{6} \times \frac{4}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} +$
 $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2}$
 $= \frac{3}{144}$

$\therefore P(A) = P_1 + P_2 = \frac{1}{360} + \frac{3}{144} = \frac{1}{45}$

$\therefore$ 恰有 4 次成功打开 4 扇门的概率为 $\frac{1}{45}$

22. 【必做题】

(1) X 取值: 1, 2, 3, 4.

$$P(X=1) = \frac{\frac{4}{15}}{\frac{4}{15} + \frac{6}{15} + \frac{8}{15} + \frac{4}{15}} = \frac{4}{15}$$
$$P(X=2) = \frac{\frac{6}{15}}{\frac{4}{15} + \frac{6}{15} + \frac{8}{15} + \frac{4}{15}} = \frac{6}{15}$$
$$P(X=3) = \frac{\frac{8}{15}}{\frac{4}{15} + \frac{6}{15} + \frac{8}{15} + \frac{4}{15}} = \frac{8}{15}$$
$$P(X=4) = \frac{\frac{4}{15}}{\frac{4}{15} + \frac{6}{15} + \frac{8}{15} + \frac{4}{15}} = \frac{4}{15}$$

X	1	2	3	4
$P(X)$	$\frac{4}{15}$	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{4}{15}$

$$E(X) = \frac{4}{15} + \frac{12}{15} + \frac{24}{15} + \frac{16}{15}$$
$$= \frac{56}{15}$$

(2) $P(A) = \frac{4}{15} \times \frac{4}{5}$

$$= \frac{16}{75}$$
$$= \frac{2}{15}$$

答: 甲球未开 4 扇门的情况下, 甲球未开 4 扇门的概率为 $\frac{2}{15}$.

22. 【必做题】

① $P(X=1) = \frac{1}{6}$
 $P(X=2) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$
 $P(X=3) = (\frac{5}{6})^2 \times \frac{1}{6} = \frac{25}{216}$
 $P(X=4) = (\frac{5}{6})^3 = \frac{125}{216}$

X	1	2	3	4
P(X)	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{216}$	$\frac{125}{216}$

$E(X) = \frac{1}{6} + \frac{5}{36} + \frac{25}{126} + \frac{125}{216} = \frac{671}{216}$

② 设打开一个门需时间为 X 分钟

$P(X) = \frac{1}{6} + \frac{5}{36} + \frac{25}{216} + \frac{125}{1296} = \frac{671}{1296}$

[illegible]

3、教学策略：

本题难度本来并不大，但题意的理解错误导致得分率较低，均分大约在 5.7 左右，学生对题意的理解存在严重问题和使用公式原理的正确率存在明显不足；计算准确度也不够。

解题格式规范问题也不小，大部分学生在解题过程中没有任何的文字说明或说明文字明显不够清晰，注意本次未在此点上严格扣分。建议在下一阶段的复习过程中，各校应注意

滚动练习，同时解题的训练应从学生的规范解题加以训练，提高高考时的达成度。